**ChE 391 Take Home Exam**

**Due 3/27/12 by 2 pm**

1. You are given the second order linear model

(20 pts)



Develop the adjoint equations and optimality equation for *u* to drive the system from (a,b) to (0,0) in minimum time. Determine the eigenvalues of the state and adjoint equations. Explain why the number of switches in *u* from its bounds will depend on the initial condition and can be a number larger than one. Recall that in your homework #2 problem 1 (minimum time), only one switch occurred in the solution. Why? Is this also related to the system eigenvalues?

1. Given the second order difference equation,

(20 pts)



Find a way to approximate this model as a discrete-time convolution FIR model,



where N is a large number. Try to do this analytically using manipulation of the difference equation up to *N=5*, but you can check your model by using linear regression of the step response.

1. Given a second order transfer function with gain=1 and time constant τ1 and τ2, derive

(20 pts) using a state space-transition matrix approach the corresponding second order difference equation based on a piecewise constant input *u(k)* and sampling time *Δt*.



You will need to find analytical relationships for a1, a2, b1, and b2 in terms of τ1 and τ2.

1. In optimal control of linear systems, verify that the eigenvalues of the adjoint equations

(10 pts) have the opposite sign of the state equations. Why is this a problem when we want to numerically integrate all dynamic equations (state and adjoint) in the same direction (such as when using the shooting method for solving the two point boundary value problem)?

1. Show that the Riccati matrix **P** is symmetric for the linear-quadratic control problem

(10 pts) (thanks to Richard P. for this question).

1. In comparing the optimal solution to the continuous time LQP with the discrete time

(10 pts) version (with dynamic programming), note that the effect of the weighting matrix **R** is different. In the continuous time case, there is a requirement that **R** be positive definite. However, this is not the case for discrete-time. Explain why this difference exists.

1. In slide 1 of “IMC Smith Presentation” posted on Blackboard, derive the closed-loop

(10 pts) transfer function for *Y/Ysp* in terms of *Gc\*, G,* and using block diagram algebra. Assume *D=0*.